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Planar gravitational corrections for supersymmetric gauge theories

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Abstract: In this paper we discuss the contribution of planar diagrams to gravitational F-terms for $\mathcal{N}=1$ supersymmetric gauge theories admitting large N description. We show how the planar diagrams lead to a universal contribution at the extremum of the glueball superpotential, leaving only the genus one contributions, as was previously conjectured. We also discuss the physical meaning of gravitational F-terms.

Keywords: Gauge Symmetry, Supersymmetric Effective Theories.

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1. Introduction

It was conjectured in [1] that for $\mathcal{N}=1$ U(N) theories admitting a large N description, the genus one non-planar diagrams compute mixed glueball/gravitational F-terms which upon substitution of the glueball extremum value yield non-perturbative corrections to gravitational F-terms. This was confirmed in a number of examples [2, 3].

The genus one contribution to the gravitational coupling was computed in [4] where the idea of C-deformation of the chiral ring [5] played a key role. The method used in [4] involved using worldsheet techniques as an inspiration to compute the relevant Feynman diagrams. Similar results were obtained in [6, 7] using anomaly considerations.

However in addition to the non-planar genus one contribution to gravitational F-terms, it turns out that even the planar ones, i.e. genus zero diagrams, also contribute to gravitational F-terms. In fact the one-loop planar diagrams in this context were already computed a long time ago [8]. In this paper we explain how to compute all the planar contributions to gravitational corrections. We show that they lead to a universal contribution independent of the coupling constants of the theory and consequently they are essentially irrelevant. We also discuss this result from the viewpoint of string theory, in cases where the gauge theory can be obtained on the worldvolume of the brane. That the planar contribution can be absorbed into a redefinition of glueball fields was also noted in [6].

The organization of this paper is as follows. In section 2 we present the string inspired computation of the gravitational corrections. In section 3 we check this result for a particular example, in the context of more conventional field theory computations. In section 4 we discuss why the planar contributions sum up to a universal term. We also discuss the physical interpretation of gravitational corrections to F-terms.

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2. String inspired computation

As in the papers [5, 4], we start our discussion on the string worldsheet. The primary goal is to understand F-terms of $\mathcal{N}=1$ gauge theories in a gravitational background. As we will see below, the string theoretical approach simplifies the computation since it automatically sums over Feynman diagrams of a given topology. The situation is somewhat similar to the one in [9] where string theory techniques were used to simplify gauge theory loop computations. However in our case the relation between gauge theories and string theory is more direct.

As shown in [10, 11], F-terms of a low energy effective theory of the type II superstring compactified on a Calabi-Yau three-fold M with or without D-branes are equal to partition functions of topological string theory. This allows us to compute F-terms of a large class of gauge theories obtained as limits of string theory. Moreover, since the string theoretical discussion that follows does not deal with details of field contents and their interactions — these are encoded in the choice of the Calabi-Yau space and of the brane configurations in it which do not show up explicitly in the discussion below — the results can be applied to any $\mathcal{N}=1$ gauge theory, provided the gauge group is $\mathrm{U}(N)$ and all the fields are in the adjoint (or fundamental) representations.

The original derivation of [10, 11] was based on the RNS formalism. A more economical derivation was given in [12] using the covariant quantization of the superstring developed in [13]. In the formalism of [13] , the four-dimensional part of the worldsheet lagrangian density that is relevant for our discussion is simply given by

$$\mathcal{L}=\frac{1}{2}\partial X^{\mu}\bar{\partial}X_{\mu}+p_{\alpha}\bar{\partial}\theta^{\alpha}+p_{\dot{\alpha}}\bar{\partial}\theta^{\dot{\alpha}}+\bar{p}_{\alpha}\partial\bar{\theta}^{\alpha}+\bar{p}_{\dot{\alpha}}\partial\bar{\theta}^{\dot{\alpha}}\,, \tag{2.1}$$

where p's are $(1,0)$-forms, $\bar{p}$'s are $(0,1)$-forms, and $\theta,\bar{\theta}$'s are 0-forms. The remainder of the lagrangian density consists of the topologically twisted $\mathcal{N}=2$ supersymmetric sigma-model on the Calabi-Yau three-fold and a chiral boson which is needed to construct the R current. We work in the chiral representation of supersymmetry in which spacetime supercharges are given by

$$\begin{aligned} Q_{\alpha} &= \oint p_{\alpha} \\ Q_{\dot{\alpha}} &= \oint p_{\dot{\alpha}}-2i\theta^{\alpha}\partial X_{\alpha\dot{\alpha}}+\cdots\,, \end{aligned} \tag{2.2}$$

where $X_{\alpha\dot{\alpha}}=\sigma^{\mu}_{\alpha\dot{\alpha}}X_{\mu}$, and $\cdots$ in the second line represents terms containing $\theta^{\dot{\alpha}}$ and $\theta^2=\epsilon_{\alpha\beta}\theta^{\alpha}\theta^{\beta}$. The second set of supercharges $\bar{Q}_{\alpha},\bar{Q}_{\dot{\alpha}}$ is defined by replacing p,θ by $\bar{p},\bar{\theta}$. These generate the $\mathcal{N}=2$ supersymmetry in the bulk. When the worldsheet is ending on D-branes and extending into four dimensions, the boundary conditions for the worldsheet variables are given by

$$\begin{aligned} (\partial-\bar{\partial})X^{\mu} &= 0\,, \\ \theta^{\alpha} &= \bar{\theta}^{\alpha}\,, \qquad p_{\alpha}=\bar{p}_{\alpha} \end{aligned} \tag{2.3}$$

Here we assume that the boundary is located at $\mathrm{Im}\,z=0$. These boundary conditions preserve one half of the supersymmetry, generated by $Q+\bar{Q}$.

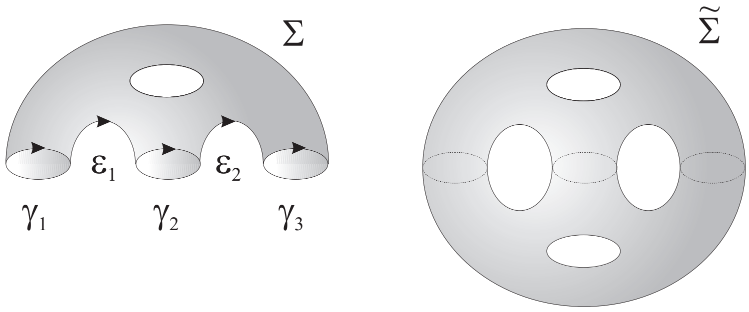

Figure 1: The open string worldsheet Σ and its double $\widetilde{\Sigma}$.

In these conventions, the vertex operators for the graviphoton $F_{\alpha\beta}$ and the gravitino $E_{\alpha\beta\gamma}$ field strengths are given by

$$\int F^{\alpha\beta} p_\alpha \bar{p}_\beta \,, \tag{2.4}$$

and

$$\int E^{\alpha\beta\gamma} \left(p_\alpha (X\bar{\partial}X)_{\beta\gamma} + \bar{p}_\alpha (X\partial X)_{\beta\gamma} + p_\alpha \bar{p}_\beta (\theta_\gamma - \bar{\theta}_\gamma) \right) \,, \tag{2.5}$$

respectively. Here $(X\partial X)_{\beta\gamma} = X_{\beta\dot{\beta}} \partial X_{\gamma\dot{\gamma}} \epsilon^{\dot{\beta}\dot{\gamma}}$. The gluino $\mathcal{W}_\alpha$ couples to the boundary γ_i of the worldsheet $(i = 1, \cdots, h)$ as

$$\oint_{\gamma_i} \mathcal{W}^\alpha (p_\alpha + \bar{p}_\alpha) \,. \tag{2.6}$$

Inserting these operators, however, is not the only effect that one has to take into account. It was pointed out in [5, 4] that, in order to preserve the $\mathcal{N} = 1$ supersymmetry, one needs modify the chiral algebra of the gluino fields so that they do not anti-commute with each other anymore. Rather they have to obey the following C-deformed relation,

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = F_{\alpha\beta} + E_{\alpha\beta\gamma} \mathcal{W}^\gamma \,. \tag{2.7}$$

To discuss the open string theory computation, it is useful to realize the worldsheet Σ of genus g with h boundaries as $\Sigma = \widetilde{\Sigma}/\mathbb{Z}_2$, where $\widetilde{\Sigma}$ is a genus $\tilde{g} = 2g + h - 1$ surface without boundary, with $\mathbb{Z}_2$ acting as the complex conjugation involution.

The boundaries of Σ are fixed point sets of the $\mathbb{Z}_2$ involution. Let us choose a basis of homology cycles of $\tilde{\Sigma}$ as $\{A_a, B_a, (a = 1, \ldots, g), \gamma_i, \epsilon_i (i = 1, \ldots, h-1)\}$, where γ_i is the cycle around the i-th boundary, ϵ_i is an interval connecting γ_i and γ_{i+1}, so that their intersections are $(A_a, B_b) = \delta_{ab}$, $(\gamma_i, \epsilon_j) = \delta_{ij} - \delta_{h,j}$ and otherwise $= 0$.

Without other operators in the bulk, we can use the Riemann bilinear identity as in [5] to rewrite the surface integral of the graviphoton vertex operator as

$$\int_\Sigma F^{\alpha\beta} p_\alpha \bar{p}_\beta = \sum_{a=1}^{g} 2F^{\alpha\beta} \oint_{A_a} (p_\alpha + \bar{p}_\alpha) \oint_{B_a} (p_\beta + \bar{p}_\beta) + \sum_{i=1}^{h-1} 2F^{\alpha\beta} \oint_{\gamma_i} (p_\alpha + \bar{p}_\alpha) \int_{\epsilon_i} (p_\beta + \bar{p}_\beta) \,. \tag{2.8}$$

Similarly the surface integral of the gravitino vertex operator can be re-expressed as

$$\int_\Sigma E^{\alpha\beta\gamma}\left(p_\alpha(X\bar\partial X)_{\beta\gamma}+\bar p_\alpha(X\partial X)_{\beta\gamma}+p_\alpha\bar p_\beta(\theta_\gamma-\bar\theta_\gamma)\right)=$$
$$=\sum_{a=1}^{g}E^{\alpha\beta\gamma}\oint_{A_a}(p_\alpha+\bar p_\alpha)\oint_{B_a}\left((X\partial X)_{\beta\gamma}-(X\bar\partial X)_{\beta\gamma}+p_\beta\theta_\gamma-\bar p_\beta\bar\theta_\gamma\right)+$$
$$+\sum_{a=1}^{g}E^{\alpha\beta\gamma}\oint_{B_a}(p_\alpha+\bar p_\alpha)\oint_{A_a}\left((X\partial X)_{\beta\gamma}-(X\bar\partial X)_{\beta\gamma}+p_\beta\theta_\gamma-\bar p_\beta\bar\theta_\gamma\right)+$$
$$+\sum_{i=1}^{h-1}E^{\alpha\beta\gamma}\oint_{\gamma_i}(p_\alpha+\bar p_\alpha)\int_{\epsilon_i}\left((X\partial X)_{\beta\gamma}-(X\bar\partial X)_{\beta\gamma}+p_\beta\theta_\gamma-\bar p_\beta\bar\theta_\gamma\right). \qquad (2.9)$$

Note that we do not have terms coming from exchanging γ_i and ϵ_i in the last line since $\partial X=\bar\partial X$ and $p=\bar p$, $\theta=\bar\theta$ on the boundaries. These terms remain if we have the gluino vertex operator (2.6) on the boundary since it has non-zero correlations with θ and $\bar\theta$ in the gravitino vertex operator. However this effect is cancelled if we turn on the C-deformation (2.7).

We are interested in terms of the form E^2S^{h-2} with $S=\mathrm{Tr}\,\mathcal{W}_\alpha\mathcal{W}^\alpha$. Let us analyze planar diagrams (so $g=0$) with h boundaries. On a planar diagram there are no A_i,B_i cycles and so the integral of the gravitino vertex is given by

$$\sum_{i=1}^{h-1}E^{\alpha\beta\gamma}\oint_{\gamma_i}(p_\alpha+\bar p_\alpha)\int_{\epsilon_i}\left((X\partial X)_{\beta\gamma}-(X\bar\partial X)_{\beta\gamma}+p_\beta\theta_\gamma-\bar p_\beta\bar\theta_\gamma\right). \qquad (2.10)$$

Note that we have the same factor $\oint_{\gamma_i}(p_\alpha+\bar p_\alpha)$ as in the gluino vertex (2.6) . The difference is that, whereas the gluino vertex operators carries an additional group theoretical factor, the gravitino vertex includes the integral along the interval ϵ_i,

$$M_{\beta\gamma}=\int_{\epsilon_i}\left((X\partial X)_{\beta\gamma}-(X\bar\partial X)_{\beta\gamma}+p_\beta\theta_\gamma-\bar p_\beta\bar\theta_\gamma\right). \qquad (2.11)$$

It acts as a generator of Lorentz transformations on the open string connecting the two boundaries γ_i and γ_{i+1}. (The Lorentz generator $M_{\mu\nu}$, which is antisymmetric in $\mu,\nu=0,\cdots,3$, can be decomposed as $M_{\alpha\dot\alpha\beta\dot\beta}=M_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}+M_{\dot\alpha\dot\beta}\epsilon_{\alpha\beta}$. We can identify $M_{\alpha\beta}$ in this decomposition as the operator (2.11).)

At this stage, it is useful to compare the computation of the E^2S^{h-2} term with the S^{h-1} term coming from the same surface with h boundaries [1, 14]: The latter is the standard superpotential term in the $\mathcal{N}=1$ gauge theory, while the former is its gravitational correction. In both cases, the $(h-1)$ zero modes of p_α are absorbed by $\oint_{\gamma_i}p_\alpha$ $(i=1,\cdots,h-1)$ in the vertex operators. For the S^{h-1} term, after absorbing the fermion zero modes, we are left with taking traces over gauge group indices around h boundaries; $(h-1)$ of them give a factor S and the remaining one gives a factor of N, the rank of the gauge group. In addition, there is a combinatorial factor of h due to the choice of one out of the h boundaries where we do not insert the gluino.

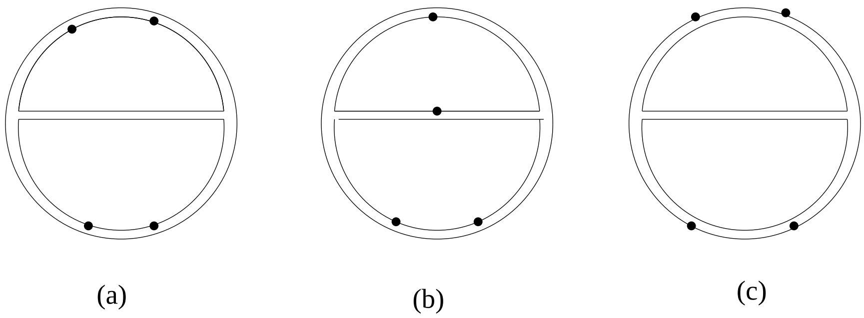

Figure 2: Two-loop diagrams for SSYM.

For the E^2S^{h-2}, there will be $(h-2)$ boundaries on which we have two gluino insertions each. In addition, we have two insertions of the Lorentz generator $M_{\alpha\beta}$ defined by (2.11) . The operator product singularities between the X's in the two $M_{\alpha\beta}$ are cancelled by those between p and θ. (This of course should have been the case due to the topological nature of the worldsheet theory.) Moreover, the zero modes of p have already been absorbed. So, the computation reduces to an integral over the momentum zero modes of X. Due to its topological nature, the computation is essentially the same as the one for the one loop case as in [15], and produces the contraction of $E_{\alpha\beta\gamma}E^{\alpha\beta\gamma}$. In addition, there is a factor of N^2 coming from the gauge group trace over the two boundaries and $h(h-1)$ due to the choice of these two boundaries. Therefore, while the standard superpotential for S takes the form $N\partial\mathcal{F}_0/\partial S$, the gravitational correction takes the form

$$\mathcal{L} = E_{\alpha\beta\gamma}E^{\alpha\beta\gamma}N^2\frac{\partial^2\mathcal{F}_0}{\partial S^2}, .$$

More generally, if we have various different boundary types where the gauge group is broken as

$$\mathrm{U}(N) \to \mathrm{U}(N_1) \times \cdots \times \mathrm{U}(N_k)\,,$$

the same reasoning as above shows that we obtain for the gravitational correction

$$\mathcal{L} = E_{\alpha\beta\gamma}E^{\alpha\beta\gamma}\sum_{i,j} N_i N_j \frac{\partial^2\mathcal{F}_0}{\partial S_j \partial S_j}\,. \tag{2.12}$$

It is useful to compare the string theoretical computation here to a field theoretical computation. Needless to say all the steps here could be given a field theoretic flavor simply by considering the $\alpha' \to 0$ version of the same arguments, though it would be cumbersome. The especially non-trivial fact is the use of the Riemann bilinear identity in organizing the sum of various field theory diagrams into a simple expression. At any rate it would be useful to check, at least in some examples, these results with those of more conventional field theoretic techniques. This we will do for a non-trivial two loop computation in the next section. As we will see, the field theoretical computation has two ingredients: one is an effect due to explicit insertions of gravity vertices (figure 3) and another is due to the gravitational C-deformation $\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = E_{\alpha\beta\gamma}\mathcal{W}^\gamma$ in diagrams involving only insertions of the Yang-Mills fields (figure 2c).

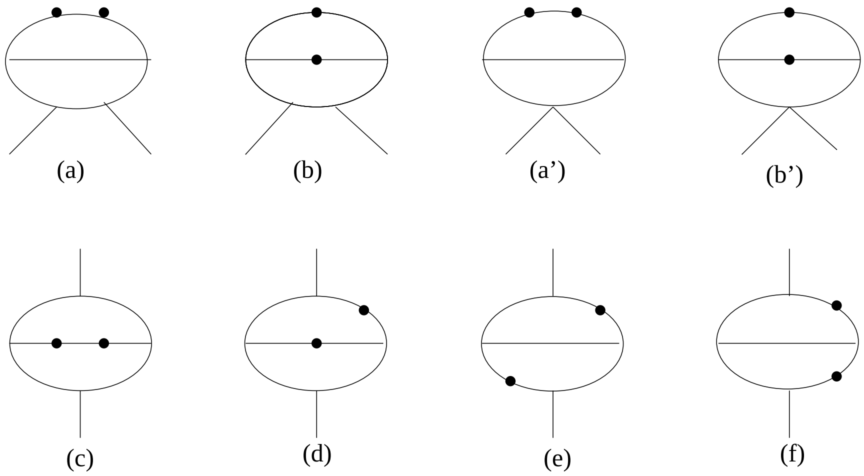

Figure 3: Two-loop diagrams for the supergravity-SSYM system.

Let us explain how these ingredients also arise in the string theory computation. One starts with the vertex operator for the gravitino (2.5), but one has to combine it with the C-deformation, which is necessary for preservation of supersymmetry due to the fermionic part of the vertex [5], in order to be able to write its surface integral as the sum of contour integrals as in (2.9). This is then used, in the case of $g = 0$, to arrive at the expression given above. Note that it is important that the gravitino vertex (2.5) has two types of terms, one of the form $pX\partial X$ and another of the form $p\bar{p}(\theta - \bar{\theta})$. It is the cancellation of the effects from these two types that maintains the topological BRST invariance on the worldsheet. For example, there is no operator product singularity between two gravitino operators of this type since the singularity coming from contractions of X's is cancelled by that coming from contractions of p and θ. One can also see, in the field theory limit, that the $pX\partial X$ term contributes in the Feynman diagrams involving explicit insertions of gravitino vertices as in figure 3, while the $p\bar{p}(\theta - \bar{\theta})$ term gives rise to the C-deformation and therefore is responsible for diagrams such as figure 2c. In the string theory computation, these two effects are combined into the single expression (2.10), from which we can read off the final result directly.

3. Planar two-loop calculation

In ref. [14] the effective glueball superpotential was computed in a perturbative field theory calculation, which led to a justification of the conjecture in [1]. The model considered there, which incorporates all the relevant features, consists of a chiral matter superfield in the adjoint representation with action

$$S = \mathrm{Tr}\left[\int d^4x d^4\theta\ \bar{\Phi}\Phi + \int d^4x d^2\theta \left(\frac{m}{2}\Phi^2 + \frac{\lambda}{3!}\Phi^3\right) + \text{h.c.}\right]. \tag{3.1}$$

The coupling to the background Yang-Mills (YM) superfield was achieved by requiring Φ to be *covariantly* chiral, i.e. $\bar{\nabla}_{\dot{\alpha}}\Phi = 0$, where $\bar{\nabla}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} - i\bar{\Gamma}_{\dot{\alpha}}$ is a derivative covariantized through the superspace YM connection $\bar{\Gamma}_{\dot{\alpha}}$.

It was shown there that order by order in perturbation theory one can integrate out the matter fields and compute the corresponding contribution to the superpotential of the gluino condensate. It is obtained from planar graphs and takes the form, at L loops, $(\mathrm{Tr}\,\mathcal{W}^2)^L$ where the Yang-Mills superspace field strength $\mathcal{W}_\alpha$ is evaluated at zero momentum.

Here we want to show that similar techniques can be employed when the matter fields are coupled to a supergravity background as well. We do this explicitly at two loops for planar index graphs and show that the perturbative field theory computation reproduces what is expected from the string theory approach of the previous section. Namely, we obtain a result proportional to $\mathcal{W}^2E^2$ where $E^2 = \frac{1}{2}E^{\alpha\beta\gamma}E_{\alpha\beta\gamma}$. (It turns out that in order to obtain this result in a standard field theory calculation it is crucial to consider a *nonabelian* YM background. This is done in order to implement the C-deformation of the chiral ring [5, 4] in the context of conventional field theory computations.) There are two rather distinct sources for the $\mathcal{W}^2E^2$ contributions and we consider them in turn.

The presence of supergravity adds new features to the calculations of ref. [14] where it was argued that only the first two diagrams of figure 2, drawn in 't Hooft double line notation with dots indicating insertions of $\mathcal{W}_\alpha$ factors, contribute to the $\int d^2\theta(\mathrm{Tr}\,\mathcal{W}^2)^2$ superpotential. For such a contribution it sufficed to consider objects in the chiral ring, i.e. objects which are annihilated by the $\bar{\nabla}_{\dot\alpha}$ spinor derivative modulo local and gauge invariant $\bar{\nabla}$-exact terms, which would not contribute to the chiral integral. In the absence of supergravity, because of the chiral ring relation

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = 0 \bmod \bar{\nabla} \tag{3.2}$$

which follows from $\{\bar{\nabla}^{\dot\alpha}, [\nabla_{\alpha\dot\alpha}, \mathcal{W}_\beta]\} = -2\{\mathcal{W}_\alpha, \mathcal{W}_\beta\}$, the only relevant object was the gluino condensate $\mathrm{Tr}\,\mathcal{W}^2$ while higher traces (in particular $\mathrm{Tr}\,\mathcal{W}^4$) vanish. This implied that not more than one pair of $\mathcal{W}_\alpha$ could be inserted in a given index loop. However, with supergravity present this is no longer the case [4], as we will now review.

From the algebra of superspace covariant derivatives satisfying the usual constraints, and as a consequence of the Bianchi identities one finds

$$[\bar{\nabla}_{\dot\alpha}, \nabla_{\beta\dot\beta}] = C_{\dot\alpha\dot\beta}\mathcal{W}_\beta + C_{\dot\alpha\dot\beta}E_{\beta\gamma}{}^{\delta}M_\delta{}^{\gamma}\,, \tag{3.3}$$

where $M_\delta{}^\gamma$ is the Lorentz generator we have introduced in the previous section, with $[M_{\alpha\beta}, \psi_\gamma] = \frac{1}{2}C_{\gamma\alpha}\psi_\beta + \frac{1}{2}C_{\gamma\beta}\psi_\alpha$ and $C_{\alpha\beta} = i\epsilon_{\alpha\beta}$ ($C^{\alpha\beta}C_{\alpha\beta} = 2$ and similarly for dotted indices). One derives then, using the Jacobi identity and the chirality of $\mathcal{W}_\alpha$

$$\begin{aligned}\{\bar{\nabla}^{\dot\alpha}, [\nabla_{\alpha\dot\alpha}, \mathcal{W}_\beta]\} &= \{[\bar{\nabla}^{\dot\alpha}, \nabla_{\alpha\dot\alpha}], \mathcal{W}_\beta\}\\ &= -2\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} - 2\{E_{\alpha\gamma}{}^{\delta}M_\delta{}^{\gamma}, \mathcal{W}_\beta\}\\ &= -2\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} - 2E_{\alpha\beta}{}^{\gamma}\mathcal{W}_\gamma\,.\end{aligned} \tag{3.4}$$

Therefore, in the presence of background supergravity, one has

$$\{\mathcal{W}_\alpha, \mathcal{W}_\beta\} = E_{\alpha\beta\gamma}\mathcal{W}^\gamma \bmod \bar{\nabla}\,. \tag{3.5}$$

We emphasize that in order to obtain a modified chiral ring relation as in (3.5) it is crucial to consider a *nonabelian* YM background. In this case no special deformation is required. Otherwise the implementation of the C-deformation would have to be realized in an unconventional fashion in field theory diagrams by introducing suitable boundary terms as in [5].

It is easy to show from (3.5) that

$$\mathcal{W}^\alpha \mathcal{W}_\alpha \mathcal{W}^\beta \mathcal{W}_\beta = -\frac{1}{12} E^{\alpha\beta\gamma} E_{\alpha\beta\gamma} \mathcal{W}^\rho \mathcal{W}_\rho \,. \tag{3.6}$$

In particular, if the computation of a Feynman amplitude produces the trace of the left-hand-side above, we can replace the result by the corresponding trace of the right-hand-side. Indeed, figure 2c with four $\mathcal{W}$'s inserted in the same index loop gives rise to such a contribution. Therefore, at the two-loop level, in addition to the previously computed $(\mathrm{Tr}\,\mathcal{W}^2)^2$, we will obtain a term $\mathrm{Tr}\,\mathcal{W}^4 \sim E^2\ \mathrm{Tr}\,\mathcal{W}^2$

We consider therefore the diagram in figure 2c. As in [14] the calculation is performed using a Schwinger parametrization for the propagators

$$\langle \Phi\Phi \rangle_i = \int_0^\infty ds_i\ \exp\left[-s_i\left(p_i^2 + \mathcal{W}^\alpha \pi_{i\alpha} + m\right)\right] . \tag{3.7}$$

Here we have set to zero the explicit supergravity dependence since it does not enter this part of the calculation; the coupling to supergravity is through the covariant derivatives in terms of which the superfield strength $\mathcal{W}_\alpha$ is defined. Also we went to Fourier transforms with respect to both space-time and spinor derivatives, introducing thus the corresponding momentum operators p and π. Finally, we have set $\bar{m} = 1$ since, by holomorphy arguments, one knows that it does not enter the final result. The actual supergraph manipulation is essentially the same as for figure 2a and can be found in [14]. With a labeling (s, t, u) for the three Schwinger parameters, after taking into account factors for combinatorics, (1/2), and group theory, $(2N^2)$, a factor of $(1/2)^2$ from the second order expansion of exponentials and a $1/3$ from a symmetrization over the Schwinger parameters, we find a net contribution

$$\begin{aligned} &\frac{N^2}{12}\,\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha \mathcal{W}^\beta \mathcal{W}_\beta)\frac{s^2t^2 + t^2u^2 + u^2s^2}{\Delta^2} = \\ &\qquad = -\frac{N^2}{144} E^{\alpha\beta\gamma} E_{\alpha\beta\gamma}\,\mathrm{Tr}(\mathcal{W}^\delta \mathcal{W}_\delta)\frac{s^2t^2 + \text{permutations}}{\Delta^2}\,, \end{aligned} \tag{3.8}$$

where

$$\Delta = st + su + ut\,. \tag{3.9}$$

This is to be multiplied by $e^{-m(s+t+u)}$ and integrated over the Schwinger parameters.

We turn now to the calculation of supergraphs with explicit insertions of supergravity vertices. As in citeDGLVZ we make use of a background covariant formulation of the theory, extended to the case of background supergravity [16, 17]. This allows us to perform the Feynman diagram computation using covariant supergraph rules which simplify the algebra in a drastic manner. We start again with the action in (3.1) where now the chiral superfield is covariantly chiral with respect to both Yang-Mills and supergravity, i.e. the spinor derivatives ∇_α and $\bar{\nabla}_{\dot\alpha}$ are covariantized with respect to both. We assume that the background is on shell. As in [14] corrections to the superpotential are obtained by computing $\int d^2\theta$ terms from vacuum diagrams with quantum vertices

$$\frac{\lambda}{3!}\int d^4x d^2\theta\ \Phi^3 \tag{3.10}$$

from the action in (3.1) and propagators

$$\langle\Phi\Phi\rangle = -\frac{\bar{m}}{\Box_+ - m\bar{m}}\,. \tag{3.11}$$

The dependence on the external fields is contained in

$$\Box_+ = \frac{1}{2}\nabla^a\nabla_a - i\mathcal{W}^\alpha\nabla_\alpha\,, \tag{3.12}$$

where $\nabla_a = -i\{\nabla_\alpha, \bar{\nabla}_{\dot\alpha}\} = E_a{}^M D_M +$ connections, $M = \{m, \mu, \dot\mu\}$, and $E_A{}^M$ is the supergravity vielbein.

In [17] we have shown that $\frac{1}{2}\nabla^a\nabla_a = \frac{1}{2}E^{aM}D_M E_a{}^N D_N$ can be expanded with respect to spinor derivatives so as to take the form

$$\frac{1}{2}\nabla^a\nabla_a = \frac{1}{2}D^a D_a - A^\alpha\nabla_\alpha - \bar{A}^{\dot\alpha}\bar{\nabla}_{\dot\alpha} - B\nabla^2 - \bar{B}\bar{\nabla}^2 - C^{\alpha\dot\alpha}[\nabla_\alpha, \bar{\nabla}_{\dot\alpha}]\,, \tag{3.13}$$

where, with the supergravity fields on shell,

$$\begin{aligned} A^\gamma &= e^{a\gamma}D_a - (D^a e_a{}^\gamma)\,, \qquad \bar{A}^{\dot\gamma} = e^{a\dot\gamma}D_a - (D^a e_a{}^{\dot\gamma}) \\ B &= \frac{1}{2}e^{a\gamma}e_{a\gamma}\,, \qquad \bar{B} = \frac{1}{2}e^{a\dot\gamma}e_{a\dot\gamma}\,, \qquad C^{\gamma\dot\gamma} = \frac{1}{2}e^{a\gamma}e_a{}^{\dot\gamma}\,. \end{aligned} \tag{3.14}$$

(However, all the terms containing $\bar{\nabla}_{\dot\alpha}$ do not contribute here and will be dropped henceforth.) As explained in [17], D_a is a space-time "covariant" derivative. At $\theta = 0$ and in Wess-Zumino gauge it reduces to the ordinary gravitational covariant derivative. The superfield $e^{a\gamma}$ is the basic object we work with (not to be confused with the vector-spinor part of the original vielbein E^{aM}, although the two are equal at the linearized level); its first component is the gravitino field.

When computing vacuum diagrams with vertices from (3.10) and propagators in (3.11), the background dependence is obtained by expanding the propagators. In this way one produces factors of ∇_α which are needed to complete the covariant D-algebra at every loop through the rule

$$\delta^{(2)}(\theta-\theta')\nabla^2\delta^{(2)}(\theta-\theta') = 1\,. \tag{3.15}$$

The external YM fields are contained in the explicit superfield strength $\mathcal{W}_\alpha$, while the relevant supergravity fields appear through terms in (3.14). Although these vertices are not in covariant form, the invariance of the action under general coordinate and local supersymmetry transformations (at the linearized level, we have invariance under the gauge transformations $\delta e_a{}^\gamma = \partial_a K^\gamma$ [17]) guarantees that the final result of our calculation will be expressible (on shell) in terms of the field strength $D_a e_b{}^\gamma - D_b e_a{}^\gamma$. We note here the relation

$$D_{[a}e_{b]\gamma} = iC_{\dot\beta\dot\alpha}E_{\alpha\beta\gamma} + iC_{\beta\alpha}E_{\dot\alpha\dot\beta\dot\gamma}\,. \tag{3.16}$$

The noncovariance of the supergravity vertices makes the supergraph calculation rather complicated. In particular, unlike the YM case where we could from the very beginning set the momenta of the external fields $\mathcal{W}_\alpha$ to zero, here, since the couplings to supergravity are proportional to the "potential" $e_a{}^\gamma$ rather than the field strength $D_{[b}e_{a]}{}^\gamma$ we cannot set immediately the gravitational external field to zero momentum. A brief description of the steps required is as follows:

a) As in the SSYM case we can carry out the rather trivial D-algebra on the supergraphs, but the complications arise from the presence of momentum factors in the numerator of the resulting Feynman integrals.

b) Using gauge invariance we project out, on each diagram, a part which is sufficient for reconstructing the full result which involves now, in the numerator, scalar products of the loop momenta. At this stage we can set the external momenta to zero.

c) After writing the propagators in exponential, Schwinger parameter, form we replace these scalar products with derivatives with respect to the parameters, after which the momentum integrals are easily carried out leaving us with a standard expression $\Delta(s_i)^{-2}e^{-m\sum s_i}$ multiplied by some additional dependence on the s_i.

We give now some details. At the two-loop level we are working with supergraphs with a total of four spinor ∇_α derivatives obtained from the expansion of the three propagators using (3.11)–(3.13), with two Yang-Mills field strengths $\mathcal{W}^\alpha\nabla_\alpha$ and one or two insertions of supergravity fields $B\nabla^2$ and $A^\alpha\nabla_\alpha$ respectively. The various supergraphs are described by the diagrams in figure 2 where the dots indicate Yang-Mills insertions and the lines supergravity insertions. These insertions produce the necessary number of spinor derivatives for carrying out the trivial D-algebra.

The YM fields are already in covariant form while the supergravity fields are not. Therefore when computing the various Feynman diagrams, we have to do the momentum integration with insertions at zero momentum for the Yang-Mills fields, but at nonzero momentum q for the supergravity fields. However, in momentum space, the final supergravity covariant result must take the form

$$\begin{aligned}\Gamma &= e^{a\gamma}(-q)(q^2\delta_{ab} - q_aq_b)e^b{}_\gamma(q)G(q^2) = \frac{1}{2}(e^{a\gamma}q^b - e^{b\gamma}q^a)(e_{a\gamma}q_b - e_{b\gamma}q_a)G(q^2)\\ &= \frac{1}{2}C^{\dot\beta\dot\alpha}E^{\alpha\beta\gamma}C_{\dot\beta\dot\alpha}E_{\alpha\beta\gamma}G(q^2) = E^{\alpha\beta\gamma}E_{\alpha\beta\gamma}G(q^2)\end{aligned} \tag{3.17}$$

and, having extracted now sufficient momentum dependence, we need only $G(0)$. (Since we are dealing with massive propagators G is nonsingular at zero momentum). Furthermore, it is only necessary to calculate, diagram by diagram, contributions proportional to $q^2\delta_{ab}$. It is then evident, looking at the structure of A^α and B in (3.14) that we only need consider the term $e^{a\gamma}D_a\nabla_\gamma$. The various possibilities are described then by the diagrams in figure 3 with (a'), (b') omitted. We parametrize the momentum dependence of the diagrams in the

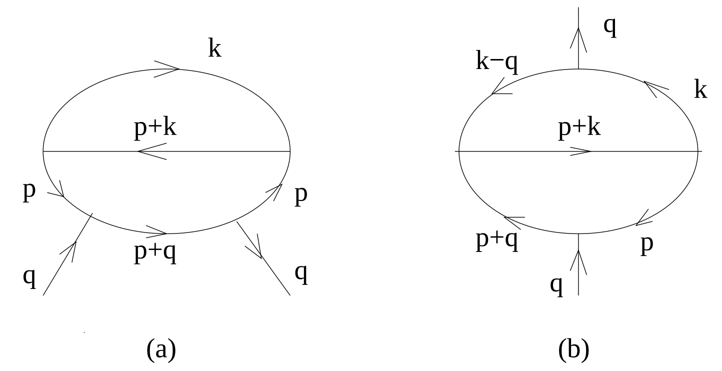

Figure 4: Momentum routing.

manner shown in figure 4. For example, figure 3a leads to the following Feynman integral:

$$\int d^4p d^4k \frac{(2p+q)_a(2p-q)_b}{(p^2+m)^2[(p+q)^2+m][(p+k)^2+m](k^2+m)^3} \,. \tag{3.18}$$

We can drop the q factors since terms like $p_a q_b$ and $q_a q_b$ will never produce, after momentum integration, a result proportional to δ_{ab}. However, the momentum integration from $p_a p_b$ and $p_a k_b$ will give contributions to both δ_{ab} and $q_a q_b$. To isolate the δ_{ab} contribution we introduce therefore the operator

$$\mathcal{O}_{ab} = 2\frac{\partial^2}{\partial q^a \partial q^b} - 5\frac{\partial^2}{\partial q^c \partial q_c}\delta_{ab} \tag{3.19}$$

which is such that $\mathcal{O}_{ab} q^a q^b = 0$. Thus, if a particular diagram produces, after momentum integration, an expression of the form

$$I^{(n)}_{ab} = q^2 \delta_{ab} G^{(n)}_1(q^2) + q_a q_b G^{(n)}_2(q^2) \tag{3.20}$$

we obtain *at zero momentum*

$$\mathcal{O}^{ab} I^{(n)}_{ab}|_{q^2=0} = -144 G^{(n)}_1(0) \tag{3.21}$$

which is all that is needed, after summing over all the diagrams, to obtain the desired result, $G(0) = \sum_1^6 G^{(n)}_1$.

We proceed in the following manner: for each diagram we use a Schwinger parameter representation of the propagators as follows:

$$\frac{1}{(p^2+m)^{n+1}} = \int_0^\infty ds\, \frac{s^n}{n!} e^{-s(p^2+m)} \,. \tag{3.22}$$

Furthermore, we find

$$\begin{aligned} \mathcal{O}_{ab} p^a p^b \left(e^{-s(p+q)^2)}\right)_{q=0} &= 12\left(3sp^2 - s^2p^4\right) e^{-sp^2} \\ \mathcal{O}_{ab} p^a k^b \left(e^{-s(p+q)^2} e^{-t(q-k)^2}\right)_{q=0} &= 4[9(s+t)p\cdot k - 3s^2p^2 k\cdot p - 3t^2k^2 p\cdot k - \\ &\quad - 2stp^2k^2 + 8st(p\cdot k)^2]e^{-sp^2-tk^2} \,. \end{aligned} \tag{3.23}$$

We note that, in general, we start with four or five Schwinger parameters. For example, for the contribution in (3.18) we must introduce separate factors

$$e^{-s_1(p^2+m)}\,, \qquad e^{-s_2[(p+q)^2+m]} \tag{3.24}$$

for the propagators on the bottom line before applying the operator $\mathcal{O}_{ab}$ and then setting $q=0$. Afterwards, part of the Schwinger parameter integration involves the integral

$$\int ds_1 ds_2 s_1 \left(3s_2 p^2 - s_2^2 p^4 e^{-(s_1+s_2)(p^2+m)}\right). \tag{3.25}$$

Changing variables to $s_1 = xs$, $s_2 = (1-x)s$ one can carry out the integration over x thus reducing the number of parameters. Other diagrams must be dealt with in a similar manner.

To carry out the momentum integrations we write

$$\begin{aligned}
&\text{(i)} \quad p.k e^{-sp^2-tk^2-u(p+k)^2} = -\frac{1}{2}\left(\frac{\partial}{\partial u} - \frac{\partial}{\partial t} - \frac{\partial}{\partial s}\right) e^{-sp^2-tk^2-u(p+k)^2} \\
&\text{(ii)} \quad p^2 e^{-sp^2-tk^2-u(p+k)^2} = -\frac{\partial}{\partial s}\left(e^{-sp^2-tk^2-u(p+k)^2}\right) \\
&\text{(iii)} \quad p^4 e^{-sp^2-tk^2-u(p+k)^2} = \frac{\partial^2}{\partial s^2}\left(e^{-sp^2-tk^2-u(p+k)^2}\right) \\
&\text{(iv)} \quad p^2 k^2 e^{-sp^2-tk^2-u(p+k)^2} = \frac{\partial^2}{\partial s \partial t}\left(e^{-sp^2-tk^2-u(p+k)^2}\right) \\
&\text{(v)} \quad p^2 (p\cdot k) e^{-sp^2-tk^2-u(p+k)^2} = \frac{1}{2}\frac{\partial}{\partial s}\left(\frac{\partial}{\partial u} - \frac{\partial}{\partial s} - \frac{\partial}{\partial t}\right)\left(e^{-sp^2-tk^2-u(p+k)^2}\right) \\
&\text{(vi)} \quad (p.k)^2 e^{-sp^2-tk^2-u(p+k)^2} = \frac{1}{4}\left(\frac{\partial}{\partial u} - \frac{\partial}{\partial s} - \frac{\partial}{\partial t}\right)^2\left(e^{-sp^2-tk^2-u(p+k)^2}\right).
\end{aligned} \tag{3.26}$$

We can perform now the momentum integration

$$\int d^4p d^4k \, e^{-sp^2-tk^2-u(p+k)^2} = \left(\frac{1}{4\pi}\right)^4 \frac{1}{[st+tu+us]^2} \tag{3.27}$$

after which we can carry out the differentiations with respect to the Schwinger parameters leading to the following individual diagram contributions:

$$\begin{aligned}
&\text{figure } 3a: \quad 4\frac{s^4t^4 + 2s^4t^3u + s^4t^2u^2 + 2s^3t^3u^2 + \text{permutations}}{\Delta^4} \\
&\text{figure } 3b: \quad \frac{2s^4t^3u + 4s^4t^2u^2 + 8s^3t^3u^2 + \text{permutations}}{\Delta^4} \\
&\text{figure } 3c: \quad -2\frac{-2s^4t^3u - 6s^4t^2u^2 - 4s^3t^3u^2 + \text{permutations}}{\Delta^4} \\
&\text{figure } 3d: \quad -2\frac{-s^4t^3u - 2s^4t^2u^2 - 10s^3t^3u^2 + \text{permutations}}{\Delta^4} \\
&\text{figure } 3e: \quad \frac{-s^4t^3u - 2s^4t^2u^2 - \frac{10}{3}s^3t^3u^2 + \text{permutations}}{\Delta^4} \\
&\text{figure } 3f: \quad \frac{-s^4t^3u - 2s^4t^2u^2 - \frac{8}{3}s^3t^3u^2 + \text{permutations}}{\Delta^4}
\end{aligned} \tag{3.28}$$

all multiplied by the factor $e^{-m(s+t+u)}$.

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In the above we have included the net factor from group theory, D-algebra, and combinatorics that each diagram contribution must be multiplied by. Additional overall factors are N^2, $(4\pi)^{-4}$ from the integration, and $-1/144$ to take into account the factor produced by the operator $\mathcal{O}_{ab}$.

Summing then all the contributions from figure 3, the result takes the form

$$\begin{aligned}
&\frac{N^2}{144(4\pi)^4}\, E^{\alpha\beta\gamma}E_{\alpha\beta\gamma}\,\mathrm{Tr}\,\mathcal{W}^\delta\mathcal{W}_\delta\int dsdtdu\; e^{-m(s+t+u)}\times\\
&\times\frac{4s^4t^4+14s^4t^3u+20s^4t^2u^2+38s^3t^3u^2+\text{permutations}}{\Delta^4}=\\
&\quad=\frac{N^2}{144(4\pi)^4}\, E^{\alpha\beta\gamma}E_{\alpha\beta\gamma}\,\mathrm{Tr}\,\mathcal{W}^\delta\mathcal{W}_\delta\int dsdtdu\; e^{-m(s+t+u)}\times\\
&\quad\times\frac{4s^2t^2+6s^2tu+\text{permutations}}{\Delta^2}\,.
\end{aligned}\tag{3.29}$$

We note that in the sum two factors of Δ have cancelled between numerator and denominator.

Finally, we add together the contributions in (3.8) and (3.29). Remarkably, just as in the pure SSYM case of [14], in the sum the denominator Δ^{-2} is cancelled and after carrying out the now trivial integral over Schwinger parameters the final result for this particular contribution takes the form

$$-\frac{N^2}{48(4\pi)^4m^3}\,E^{\alpha\beta\gamma}E_{\alpha\beta\gamma}\,\mathrm{Tr}\,\mathcal{W}^\delta\mathcal{W}_\delta \tag{3.30}$$

a result consistent with that from string theory.

4. Universality of the planar contribution

From what we have seen the mixed glueball/gravitational F-terms from genus zero takes the form

$$\mathcal{L}=E^{\alpha\beta\gamma}E_{\alpha\beta\gamma}\sum_{i,j}N_iN_j\frac{\partial^2\mathcal{F}_0}{\partial S_i\partial S_j}\,,$$

where $\mathcal{F}_0$ is the planar partition function. We should note that the full prepotential $\mathcal{F}_0$ also includes the measure factor $\frac{1}{2}\sum_i S_i^2\log S_i$ (which for the similar expression for the superpotential yields the standard Veneziano-Yankielowicz expression $\sum_i N_iS_i\log S_i$ for the gauge factors $\mathrm{U}(N_i)$). This measure factor should also be included for the gravitational contribution, where it gives the term proportional to

$$\sum_i N_i^2\log S_i\,.$$

This is a direct consequence of the gravitational contribution to the axial anomaly. Note that the contribution of each gauge factor $\mathrm{U}(N_i)$ is proportional to N_i^2, since the gravitational term in the anomaly keeps track of all the perturbative degrees of freedom that are running around in the fermion loop. In this case these are the N_i^2 components of the gluinos.

Now consider the expectation value of the gluino bilinear S_i as determined by extremization of the superpotential, which gives the equation [1]

$$N_i \partial_i \partial_j F_0 + \tau = 0\,,$$

with τ the bare gauge coupling of the $\mathrm{U}(N)$ gauge theory.

When the corresponding solutions for the S_i are substituted into the gravitational correction we have computed, one obtains therefore

$$\mathcal{L} = -E^{\alpha\beta\gamma} E_{\alpha\beta\gamma} \sum_j N_j \tau\,.$$

This correction is proportional to the universal contribution $N\tau$ where $N = \sum_j N_j$. It only depends on the total rank N of the gauge theory and is independent of the particular symmetry breaking pattern and of all the details of the $\mathcal{N} = 1$ superpotential.

In fact, it is easy to see that this contribution, proportional to $N\tau$, is also needed for the closed string dualities to work, if we embed these gauge theories into superstrings [18, 19]: For example consider Type IIB strings with some D5 branes wrapping 2-cycles of a CY and filling the spacetime. Then for each brane there is a well-known $R \wedge R$ correction on its worldvolume [20]. Since the volume of the internal part of the $D5$ brane is given by τ, this yields a term in four dimensions given by $\tau E^{\alpha\beta\gamma} E_{\alpha\beta\gamma}$. Since we have N such branes this gives exactly the contribution

$$E^{\alpha\beta\gamma} E_{\alpha\beta\gamma} N\tau\,.$$

In the context of superstrings this term comes in addition to the glueball contributions we computed above. Thus, we now see that the two effects — the induced curvature term on the brane and the sum of the planar diagrams of the gauge theory — exactly cancel out. In particular on the closed string dual, where the branes have disappeared completely, there should be no genus zero correction to $E^{\alpha\beta\gamma} E_{\alpha\beta\gamma}$; there should only be the genus one contribution. This is indeed the case [10, 11].

Note that, if we do not extremize the superpotential, the gravitational correction receives contributions from both genus zero and genus one diagrams. In cases when these diagrams can be exactly summed and give rise to an effective spectral curve, as in [21, 22], these contributions have a direct geometric interpretation. The genus zero quantity $\partial_i \partial_j \mathcal{F}_0$ gives the period matrix τ_{ij}^{eff} of the effective curve [1], and the genus one term $\mathcal{F}_1$ can be expressed as the chiral scalar determinant [2, 3]. Combining these two facts, we can write the gravitational correction as $\log Z$ with

$$Z = \frac{e^{\pi i N_i \tau_{ij}^{\mathrm{eff}} N_j}}{\sqrt{\det \Delta}}\,.$$

We note the amusing fact that Z takes the form of a holomorphic block of a chiral boson on the spectral curve with loop momenta N_i.

4.1 Gravitational F-term as domain wall partition function

As we have argued the whole non-trivial contribution of the gravitational F-terms will come from genus one diagrams, i.e.

$$\mathcal{L} = \mathcal{F}_1(S_i) E^{\alpha\beta\gamma} E_{\alpha\beta\gamma} \,,$$

where we substitute the value of S_i found from the extremization of the superpotential, as computed using the planar diagrams. It is natural to ask what is the $\mathcal{N} = 1$ supersymmetric gauge theory significance of this term. Let us think of this as if it were to come from a dual closed string theory. In this context we see that N does not enter this expression, so it would have made sense also for $\mathcal{N} = 2$ theories, where the flux, which breaks half of the supersymmetries and is proportional to N, is set to zero. In the context of $\mathcal{N} = 2$ supersymmetric theories obtained by type IIB strings on CY 3-folds, it has been argued in [23] that the genus one term $\mathcal{F}_1$ computes the partition function of BPS D3 branes wrapped over cycles of the Calabi-Yau. Roughly speaking we have

$$\mathcal{F}_1 = \frac{1}{12} \sum_{\text{BPS states}} (-1)^s \log m \,,$$

where s denotes the spin of the D3 brane state and m denotes its mass, given by $|\int_{D3} \Omega|$. This is only roughly the description because the number of D3 branes can jump over moduli whereas $\mathcal{F}_1$ is smooth. This is because $\mathcal{F}_1$ includes also contributions from multi-particle sectors of D3 branes as in [24]. It would be interesting to make this interpretation of the gravitational correction as counting BPS states more precise.

However, for the case at hand, with generically just $\mathcal{N} = 1$ supersymmetry, this is not a satisfactory interpretation of the gravitational correction, because there is no notion of BPS particle for $\mathcal{N} = 1$ supersymmetric gauge theories. The only BPS object is the domain wall. In the string setup this is related to D5 branes which wrap a 3-cycle inside a CY and are a domain wall in $\mathbb{R}^4$. Since the internal part of the counting of these BPS domain walls is the same as the counting of BPS particles in the associated $\mathcal{N} = 2$ supersymmetric theory, it is natural to conjecture that the $\mathcal{N} = 1$ supersymmetric interpretation of gravitational F-term is as a partition function of domain walls.

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